



# **RESILIENT MONITORING SYSTEM: Design and Performance Analysis**

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# Outline

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1. Introduction
2. Modeling and Problem Formulation
3. Information Assessment Layer
4. Plant Assessment Layer
5. Sensor Adaptation Layer
6. Performance of Resilient Monitoring System
7. Conclusions and Future Work



# 1. INTRODUCTION

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- **The goal:** Design and evaluate the performance of an autonomous power plant monitoring system that *degrades gracefully* under natural or malicious sensor malfunctioning. We refer to such a system as *resilient*.
- **Resilient vs. adaptive system:**
  - Adaptive systems change their behavior in response to *external* conditions.
  - Resilient systems change their behavior in response to *internal* conditions.
- **Approach:** In this work, resiliency is achieved by utilizing the so-called *rational controllers* that force the monitoring system (sensor network) operate in the state where the *entropy* of the estimated plant probability mass function is minimized.

## 2. MODELING AND PROBLEM FORMULATION

### 2.1 Sensor

#### ■ **Model:**

- $\mathbf{V}$  – process variable;  $\mathbf{S}$  – sensor assigned to monitor  $\mathbf{V}$ .
- $V$  – random variable representing state of  $\mathbf{V}$ ;  $S$  – random variable representing state of  $\mathbf{S}$ .
- The state space of  $\mathbf{V}$  and  $\mathbf{S}$  is {Low, Normal and High}, i.e.,  $\Sigma = \{L, N, H\}$
- Sensor measurement quality (*Data Quality*) – a number,  $DQ$ , between 0 and 1 provided by a “watchdog” monitoring system.
- $DQ=1$  implies data perfectly trustworthy;  $DQ=0$  implies worthless data;  $DQ \in (0, 1)$  implies a certain level of trustworthiness.
- Model of the  $DQ$  effect on  $V$  and  $S$  coupling is:
$$\begin{aligned} P\{V = \sigma \mid S = \sigma\} &= B, \\ P\{V \neq \sigma \mid S = \sigma\} &= \frac{1 - B}{2} \end{aligned} \quad \begin{array}{c} B = \frac{2}{3}DQ + \frac{1}{3} \\ \uparrow \end{array}$$

referred to as *believability*

#### ■ **Problem:**

- Estimate pmf of  $V$ , i.e.,  $\hat{P}\{V = \sigma\} := \lim_{n \rightarrow \infty} P\{V = \sigma \mid s_1, s_2, \dots, s_n; DQ\}, \sigma \in \Sigma$
- Estimate pmf of  $V$  when two or more sensors are associated with  $V$ . For example, when two sensors are present, estimate

$$\begin{aligned} \hat{P}\{V = \sigma\} := \lim_{n, m \rightarrow \infty} P\{V = \sigma \mid s_1^1, s_2^1, \dots, s_n^1; DQ_1; \\ s_1^2, s_2^2, \dots, s_m^2; DQ_2\}, \sigma \in \Sigma. \end{aligned}$$

- This constitutes the *information assessment layer* of the resilient monitoring system.



## 2.2 Plant

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- **Model:**

- $\mathbf{G}$  – plant
- $G$  – random variable representing the state of  $\mathbf{G}$ .
- The state space of  $\mathbf{G}$  is:

$$G \in \Sigma_G = \{L_G, N_G, H_G\}$$

- The plant model

$$\mathbf{G} : [P(V_1 | G), \dots, P(V_M | G)], \quad V_i \in \Sigma, \quad G \in \Sigma_G$$

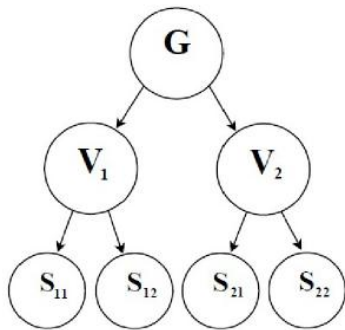
- **Problem:**

- Using  $\hat{P}(V_1), \dots, \hat{P}(V_M)$ , evaluate  $\hat{P}(G)$ ,  $G \in \Sigma_G$ .
- This constitutes the *plant assessment layer* of the resilient monitoring system.

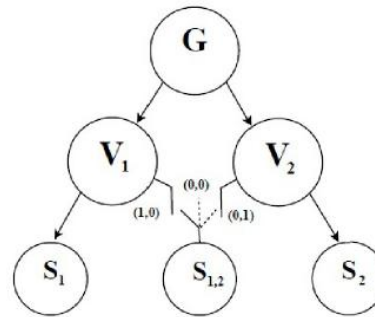
## 2.3 Sensor network and resiliency problem

### ■ Model:

- Two types of sensors: *dedicated* and *free*.
  - Dedicated sensors: measure the process variable to which they are assigned.
  - Free sensors: measure any process variable from a finite set to which they are wired.
- Two types of sensor networks: *non-contentious* and *contentious*:
  - Non-contentious case: decision to be made is whether to use or not a particular sensor for process variable pmf estimation;
  - Contentious case: in addition to above, decide which process variable a sensor should be assigned to.



(a) Non-contentious



(b) Contentious

- $X$  – sensor network state space, e.g.,  
 Non-contentious case:  $(1010) \in X$   
 Contentious case:  $(1(01)0) \in X$

### ■ Resiliency problem:

- Autonomously (i.e., without external interference) and independently (i.e., without communications among the sensors) identify and force the network operate in the state  $x^*$  such that  $I_G(x^*) = \min_{x \in X} I_G(x)$  where  $I_G(x)$  is the entropy of the plant pmf when the network is in state  $x$ :

$$I_G(x) = - \sum_{\sigma \in \Sigma_G} \hat{P}_x(G = \sigma) \log \hat{P}_x(G = \sigma).$$

- This constitutes the *adaptation layer* of the resilient monitoring system.

## 2.4 Adaptation method and measure of resiliency

### ■ Adaptation method:

- Adaptation in this work is based on *rational controllers*.
- Rational controllers are dynamical systems possessing two properties: *ergodicity* and *rationality*.
  - Ergodicity implies each state in the state space is visited with non-zero probability.
  - Rationality implies the *residence time* in states with a smaller *penalty* is larger than in those with larger penalty.
- When the so-called *measure of rationality* is large enough, rational controllers force the system operate in the state, which has the smallest penalty, with the largest probability.

### ■ Measure of resiliency:

- Introduce expected value of the estimated plant pmf:  $\tilde{P}(G) = \sum_{x \in X} \alpha_x \hat{P}_x(G)$   
where  $\alpha_x$  is the probability of plant operating in state  $x$ .
- Then, *measure of resiliency (MR)*  $MR$  is given by

$$MR = \frac{D(P(G) || \hat{P}_{nr}(G)) - D(P(G) || \tilde{P}(G))}{D(P(G) || \hat{P}_{nr}(G))},$$

where  $P(G)$  is the true plant pmf and operator  $D$  indicates the Kullback-Leibner divergence:

$$D(P(G) || \tilde{P}(G)) = \sum_{\sigma \in \Sigma_G} P\{G = \sigma\} \log \frac{P\{G = \sigma\}}{\tilde{P}\{G = \sigma\}}.$$

### ■ Problem:

- Select structure and parameters of rational controllers appropriate for the resilient monitoring system.
- Design the penalty function for the problem at hand.
- For the system, thus designed evaluate the measure of resiliency,  $MR$ .

## 2.5 Example for numerical investigation

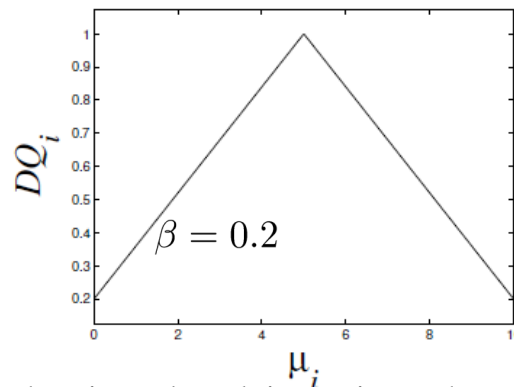
- **Process variable pmf:**  $P_L^V = \int_0^{\frac{10}{3}} f(v)dv, P_N^V = \int_{\frac{10}{3}}^{\frac{20}{3}} f(v)dv, P_H^V = \int_{\frac{20}{3}}^{10} f(v)dv$
- **Sensor measurement pmf:**

$$P_L^{S_i} = \int_0^{\frac{10}{3}} \frac{1}{\sqrt{2\pi}\sigma_{S_i}} e^{-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma_{S_i}}\right)^2} dx, \quad P_N^{S_i} = \int_{\frac{10}{3}}^{\frac{20}{3}} \frac{1}{\sqrt{2\pi}\sigma_{S_i}} e^{-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma_{S_i}}\right)^2} dx, \quad P_H^{S_i} = \int_{\frac{20}{3}}^{10} \frac{1}{\sqrt{2\pi}\sigma_{S_i}} e^{-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma_{S_i}}\right)^2} dx.$$

- **Cyber threats:** Change  $\mu_i$  of sensors away from  $E(V_{cont})$ ; sensor data quality ( $DQ$ ) is generated by a “watchdog” monitoring system.
- **Data quality model:**
  - $DQ_i=1$  if  $\mu_i = E(V_{cont})$ ; if  $\mu_i$  is away from  $E(V_{cont})$ ,  $DQ_i$  decreases according to the rule

$$DQ_i = \begin{cases} 1 - \frac{(1-\beta)|E(S_i) - E(V_{cont})|}{10 - E(V_{cont})}, & \text{if } 0 < E(V_{cont}) \leq 5, \\ 1 - \frac{(1-\beta)|E(S_i) - E(V_{cont})|}{E(V_{cont})}, & \text{if } 5 < E(V_{cont}) < 10, \end{cases} \quad \text{where } \beta \in (0, 1).$$

- Illustration for  $E(V_{cont})=5$ :



- **Plant Model:**

$$P(V_1|G) = P(V_2|G) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

- **Problem:** Using the data introduced, investigate the performance of resilient monitoring system.

### 3. INFORMATION ASSESSMENT LAYER

#### 3.1 Estimation of process variable pmf based on a single sensor

■ **The h-procedure:**

- Notation:  $h_{\sigma}(n) := P\{V = \sigma | s_1, \dots, s_n; DQ\}$ ,  $\sigma \in \Sigma$ ,  $n \in \mathbb{N}^*$ .
- Recursive procedure:

$$h_{\sigma}(n+1) = h_{\sigma}(n) + \epsilon(n)[h_{\sigma}^*(s_{n+1}) - h_{\sigma}(n)], \sigma \in \Sigma, n \in \mathbb{N}$$

where

$$0 < \epsilon(n) \leq 1, \sum_{n=1}^{\infty} \epsilon(n) = \infty, \sum_{n=1}^{\infty} \epsilon^2(n) < \infty, n \in \mathbb{N}$$

$$h_{\sigma}^*(s_{n+1}) := \begin{cases} B, & \text{if } s_{n+1} = \sigma, \\ \frac{1-B}{2}, & \text{if } s_{n+1} \neq \sigma, \end{cases} \quad B: \text{sensor believability}$$

■ **Lemma:** The expected value of the set point is given by

$$E(h_{\sigma}^*(s_n)) = DQ \cdot P\{S = \sigma\} + \frac{1 - DQ}{3}, \sigma \in \Sigma, \forall n \in \mathbb{N}.$$

■ **Theorem:** The h-procedure converges in probability to the expected value of the set point, i.e.,

$$h_{\sigma}(n) \xrightarrow{P} DQ \cdot P\{S = \sigma\} + \frac{1 - DQ}{3}, \sigma \in \Sigma, \text{ when } n \rightarrow \infty.$$

■ Thus, the process variable pmf estimate is:  $\hat{P}(V = \sigma) = DQ \cdot P\{S = \sigma\} + \frac{1 - DQ}{3}, \sigma \in \Sigma$

## 3.2 Estimation of process variable pmf based on multiple sensors

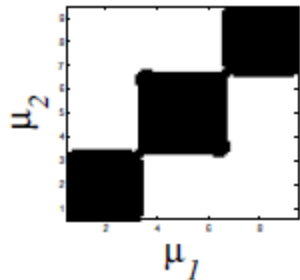
- **Dempster-Shafer combination rule:**  $\hat{P}_{S_1 S_2}\{V = \sigma\} = \frac{\hat{P}_{S_1}\{V = \sigma\} \hat{P}_{S_2}\{V = \sigma\}}{\sum_{\sigma \in \Sigma} \hat{P}_{S_1}\{V = \sigma\} \hat{P}_{S_2}\{V = \sigma\}},$

where

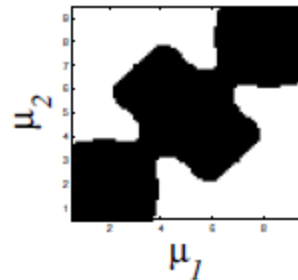
$$\begin{aligned} \hat{P}_{S_1 S_2}\{V = \sigma\} &:= P\{V = \sigma | s_1^1, \dots, s_n^1, \dots; DQ_1; \\ &\quad s_1^2, \dots, s_n^2, \dots; DQ_2\}, \sigma \in \Sigma \end{aligned} \quad \begin{aligned} \hat{P}_{S_1}\{V = \sigma\} &:= P\{V = \sigma | s_1^1, \dots, s_n^1, \dots; DQ_1\}, \\ \hat{P}_{S_2}\{V = \sigma\} &:= P\{V = \sigma | s_1^2, \dots, s_n^2, \dots; DQ_2\}. \end{aligned}$$

- **Monotonicity issue:**

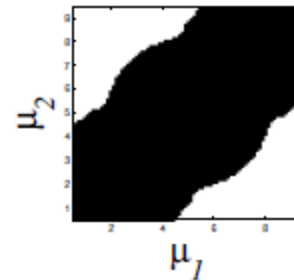
- Is the entropy of  $\hat{P}_{S_1 S_2}\{V = \sigma\}$  always smaller than the entropy of  $\hat{P}_{S_1}\{V = \sigma\}$  and  $\hat{P}_{S_2}\{V = \sigma\}$ ?  
If the answer is in the affirmative, the system is monotonic w.r.t. the number of sensors, and all sensors should be always used. If not, adaptation is necessary to select the right combination of sensors, ignoring the ones that increase entropy.
- It turns out that the system at hand is non-monotonic: The areas of non-monotonicity (white areas below) depend on the standard deviation of the sensor measurement:



(a)  $\sigma^S = 0.2$



(b)  $\sigma^S = 1$



(c)  $\sigma^S = 2$



## 4. PLANT ASSESSMENT LAYER

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### 4.1 Estimation of plant pmf using a single process variable

**Algorithm 4.1:** Given initial  $P_0 = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$

(a) Calculate  $P_0(V, G) = P_0(G)P(V|G)$

(b) Calculate  $P_0(V) = \sum_G P_0(V, G)$

(c) Calculate  $\hat{P}(V, G) = P_0(V, G) \frac{\hat{P}(V)}{P_0(V)}$  (Jeffrey's rule)

(d) Set  $\hat{P}(G) = \sum_V \hat{P}(V, G)$

## 4.2 Estimation of plant pmf using multiple process variables

- **Algorithm 4.2:** Given desired accuracy  $\Delta > 0$  for terminating iterative algorithm below and initial  $P_0(G) = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$

(a) Calculate joint probability distribution  $P_0(V_1, V_2, G)$ . Set  $i = 0$ .

(b) Calculate

$$P_i(V_1) = \sum_{V_2} \sum_G P_i(V_1, V_2, G)$$
$$P_{i+1}(V_1, V_2, G) = P_i(V_1, V_2, G) \frac{\hat{P}(V_1)}{P_i(V_1)}$$
$$i = i + 1$$

(c) Calculate

$$P_i(V_2) = \sum_{V_1} \sum_G P_i(V_1, V_2, G)$$
$$P_{i+1}(V_1, V_2, G) = P_i(V_1, V_2, G) \frac{\hat{P}(V_2)}{P_i(V_2)}$$
$$i = i + 1$$

(d) Calculate

$$P_i(V_1) = \sum_{V_2} \sum_G P_i(V_1, V_2, G) \quad \text{and} \quad P_i(V_2) = \sum_{V_1} \sum_G P_i(V_1, V_2, G)$$

If  $\|\hat{P}(V_1) - P_i(V_1)\| < \Delta$  and  $\|\hat{P}(V_2) - P_i(V_2)\| < \Delta$ , set

$$\hat{P}(G) = \sum_{V_1} \sum_{V_2} P_i(V_1, V_2, G).$$

Otherwise, return to step (b).

- This algorithm is a version of the Iterative Proportional Fitting Procedure (IPFP) and is known to converge under initial conditions indicated above.

## 5. SENSOR ADAPTATION LAYER

### 5.1 Rational controller

- Let  $\varphi(x) > 0$  be the penalty function associated with each state  $x$  in  $X$ .
- Let the residence time of the rational controller in state  $x$  be  $T(x) = \left(\frac{1}{\varphi(x)}\right)^N$ , where the positive integer  $N$  is the *measure of rationality*.
- After  $T(x)$  elapses, the controller selects a new state in  $X$  with equal probability. Clearly,  
$$\frac{T(x_i)}{T(x_j)} > 1, \text{ if } \varphi(x_i) < \varphi(x_j).$$
- Moreover,  
$$\frac{T(x_i)}{T(x_j)} \rightarrow \infty, \text{ as } N \rightarrow \infty, \text{ if } \varphi(x_i) < \varphi(x_j).$$
- Thus, this rational controller resides the longest in the state with the smallest penalty, and this property becomes more pronounced when  $N$  is large.
- Let  $\tau(x_i)$  denote the *relative residence time* in  $x_i$ , i.e.,  $\tau(x_i) = \frac{T(x_i)}{\sum_{x \in X} T(x)}$ .  
Then  
$$\tau(x_i) > \tau(x_j), \text{ if } \varphi(x_i) < \varphi(x_j).$$

### 5.2 Penalty function

- Selected as the entropy of the estimate of the plant pmf, i.e.,  
$$\varphi(x) = I_G(x), \quad x \in X$$
- The performance of the resilient monitoring system based on this penalty function is quantified next using the sensor networks introduced in Subsections 2.3 and 2.5.

# 6. PERFORMANCE OF RESILIENT MONITORING

## 6.1 Non-contentious case

### ■ Relative residence time:

(a)  $\mu_{11} = 5.4, \mu_{12} = 5.3, \mu_{21} = 4.9, \mu_{22} = 5.3; \sigma^S = 1.5$

	$N=2$	$N=5$	$N=30$	$N=50$
$\tau(1111)$	<b>0.2016</b>	<b>0.5979</b>	$\approx 1$	$\approx 1$
$\tau(1110)$	0.0843	0.0545	1.443E-6	1.75E-10
$\tau(1101)$	0.0812	0.058	7.58E-7	7.4E-11
$\tau(1011)$	0.0686	0.0582	8.98E-7	1E-10
$\tau(1010)$	0.0504	0.0175	1.13E-9	1.35E-15
$\tau(1001)$	0.0521	0.0174	1.03E-9	1.06E-15
$\tau(0111)$	0.0728	0.0583	1.81E-6	2.53E-10
$\tau(0110)$	0.0555	0.018	1.78E-9	2.33E-15
$\tau(0101)$	0.047	0.0179	1.31E-9	1.39E-15
$\tau(1100)$	0.0644	0.0274	2.22E-8	1.47E-13
$\tau(0011)$	0.0618	0.0363	5.4E-8	6.9E-13
$\tau(1000)$	0.0414	0.0072	1.38E-11	9.69E-19
$\tau(0100)$	0.0422	0.01	2.11E-11	1.66E-18
$\tau(0010)$	0.0376	0.0116	3.11E-11	3.19E-18
$\tau(0001)$	0.0346	0.0099	2.11E-11	1.78E-18

(c)  $\mu_{11} = 5.4, \mu_{12} = 8.5, \mu_{21} = 4.9, \mu_{22} = 2.3; \sigma^S = 1.5$

	$N=2$	$N=5$	$N=30$	$N=50$
$\tau(1111)$	0.0671	0.0575	0.0102	0.0007
$\tau(1110)$	0.0754	0.085	0.0526	0.0108
$\tau(1101)$	0.0517	0.0376	0.0003	1.38E-6
$\tau(1011)$	0.0823	0.1219	0.1648	0.0748
$\tau(1010)$	<b>0.0987</b>	<b>0.1424</b>	<b>0.7265</b>	<b>0.9102</b>
$\tau(1001)$	0.0608	0.0535	0.001	1.37E-5
$\tau(0111)$	0.0506	0.0396	0.0004	2.93E-6
$\tau(0110)$	0.0567	0.0495	0.001	1.26E-5
$\tau(0101)$	0.0569	0.0369	0.0002	1.16E-6
$\tau(1100)$	0.0622	0.0508	0.0029	0.0001
$\tau(0011)$	0.0776	0.0662	0.011	0.0007
$\tau(1000)$	0.0747	0.0694	0.0092	0.0007
$\tau(0100)$	0.0544	0.0542	0.001	1.55E-5
$\tau(0010)$	0.0688	0.0862	0.0177	0.002
$\tau(0001)$	0.0622	0.0511	0.0012	2.01E-5

(b)  $\mu_{11} = 5.4, \mu_{12} = 5.3, \mu_{21} = 4.9, \mu_{22} = 2.3; \sigma^S = 1.5$

	$N=2$	$N=5$	$N=30$	$N=50$
$\tau(1111)$	0.0928	0.1797	0.0985	0.0248
$\tau(1110)$	<b>0.1223</b>	<b>0.1878</b>	<b>0.887</b>	<b>0.9744</b>
$\tau(1101)$	0.0548	0.0337	1.18E-5	9.02E-9
$\tau(1011)$	0.062	0.0576	0.0002	8.59E-7
$\tau(1010)$	0.0717	0.0737	0.0008	9.52E-6
$\tau(1001)$	0.0434	0.0249	9.71E-7	1.39E-10
$\tau(0111)$	0.0712	0.0567	0.0002	1.63E-6
$\tau(0110)$	0.0793	0.074	0.0009	1.4E-5
$\tau(0101)$	0.0458	0.0252	1.26E-6	1.91E-10
$\tau(1100)$	0.1011	0.1132	0.123	0.0008
$\tau(0011)$	0.0514	0.037	1.09E-5	7.34E-9
$\tau(1000)$	0.0516	0.0327	9.48E-6	6.1E-9
$\tau(0100)$	0.0531	0.0375	1.25E-5	1.07E-8
$\tau(0010)$	0.0529	0.0406	1.82E-5	1.92E-8
$\tau(0001)$	0.0467	0.0256	1.11E-6	2.1E-10

### ■ Measure of resiliency:

$P(G) = [0 \ 1 \ 0]$			
System	$I(a)$	$I(b)$	$I(c)$
$D(P(G)  P_{nr}(G))$	0.1692	0.4036	0.7617
$D(P(G)  \hat{P}(G))$	0.1692	0.3394	0.5471
MR	$\approx 0$	0.1592	0.2818

## 6.2 Contentious case

### Relative residence time:

(a)  $\mu_1 = 5.1, \mu_2 = 5.1, \mu_{1,2} = 5.2; \sigma^S = 1.5$

	$N=2$	$N=5$	$N=30$	$N=50$
$\tau(1(10)1)$	0.1278	0.2032	0.522	0.4743
$\tau(1(01)1)$	0.1458	0.2384	0.4614	0.5248
$\tau(1(00)1)$	0.0845	0.0665	0.0003	3.28E-6
$\tau(1(01)0)$	0.0883	0.0631	0.0003	2.09E-6
$\tau(0(10)1)$	0.0793	0.0692	0.0003	2.09E-6
$\tau(1(10)0)$	0.1046	0.1199	0.008	0.0005
$\tau(0(01)1)$	0.1104	0.1137	0.0076	0.0004
$\tau(1(00)0)$	0.065	0.0339	3.95E-6	1.7E-9
$\tau(0(10)0)$	0.0678	0.0304	3.57E-6	1.39E-9
$\tau(0(00)1)$	0.0605	0.0316	4.32E-6	1.67E-9
$\tau(0(01)0)$	0.066	0.0301	2.91E-6	1.18E-9

(b)  $\mu_1 = 5.1, \mu_2 = 5.1, \mu_{1,2} = 2.5; \sigma^S = 1.5$

	$N=2$	$N=5$	$N=30$	$N=50$
$\tau(1(10)1)$	0.1149	0.1606	0.3188	0.3061
$\tau(1(01)1)$	0.1099	0.1633	0.3294	0.2991
$\tau(1(00)1)$	0.1293	0.1632	0.3316	0.3938
$\tau(1(01)0)$	0.0745	0.0522	0.0005	7.22E-6
$\tau(0(10)1)$	0.0679	0.0566	0.0004	5.79E-6
$\tau(1(10)0)$	0.0957	0.0766	0.0047	0.0003
$\tau(0(01)1)$	0.0883	0.0732	0.005	0.0003
$\tau(1(00)0)$	0.0791	0.0783	0.0043	0.0002
$\tau(0(10)0)$	0.0714	0.0493	0.0003	2.03E-6
$\tau(0(00)1)$	0.0868	0.0766	0.0048	2E-4
$\tau(0(01)0)$	0.0821	0.0501	0.0003	2.14E-6

(c)  $\mu_1 = 5.1, \mu_2 = 2.5, \mu_{1,2} = 5.1; \sigma^S = 1.5$

	$N=2$	$N=5$	$N=30$	$N=50$
$\tau(1(10)1)$	0.0962	0.0741	0.0007	9.18E-6
$\tau(1(01)1)$	0.1054	0.1319	0.0308	0.0044
$\tau(1(00)1)$	0.075	0.0471	4.66E-5	9.16E-8
$\tau(1(01)0)$	0.1024	0.1293	0.0374	0.0053
$\tau(0(10)1)$	0.0782	0.0426	4.58E-5	8.54E-8
$\tau(1(10)0)$	0.1414	0.2553	0.9294	0.9903
$\tau(0(01)1)$	0.0832	0.0723	0.0004	4.04E-6
$\tau(1(00)0)$	0.0879	0.0686	0.0004	3.03E-6
$\tau(0(10)0)$	0.0821	0.0705	0.0004	3.13E-6
$\tau(0(00)1)$	0.0647	0.0404	2.91E-5	2.9E-8
$\tau(0(01)0)$	0.0833	0.0679	0.0004	3.32E-6

### Measure of resiliency:

$$P(G) = [0 \ 1 \ 0]$$

System	$II(a)$	$II(b)$	$II(c)$
$D(P(G)  P_{nr}(G))$	0.3226	0.5158	0.6303
$D(P(G)  \hat{P}(G))$	0.3228	0.5126	0.4112
$MR$	-6.36E-4	0.006	0.3477



## 7. CONCLUSIONS AND FUTURE WORK

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- **This work demonstrated the following:**
  - The model of data quality introduced may be used for designing monitoring systems.
  - The h-procedure can be used for process variable pmf accommodation under non-perfect data quality.
  - The Dempster-Shafer combination rule and Kullback-Leibner divergence are useful for pmf and level of resiliency evaluation.
  - Rational controllers and entropy-based penalty function are applicable to the problem at hand.
  - The three-level monitoring systems is an appropriate architecture for designing resilient monitoring systems for plant health assessment applications.
  - This work demonstrated that the behavior of the resilient monitoring system is, in most, cases, akin the performance of a human operator.



## 7. CONCLUSIONS AND FUTURE WORK (cont)

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- **Several topics for future work remain open, including:**
  - As far as the system designed is concerned, more extensive numerical evaluation should be carried out, including
    - dynamical (temporal) properties of resilient adaptation;
    - resilient behavior for more realistic plant models and sensor networks.
  - Future work includes:
    - assessment interpretation and calculation methods of plant health;
    - investigation of more efficient structures of rational controllers;
    - design of more appropriate penalty functions for effective resiliency;
    - development of more effective methods for process variable and plant state pmf's estimation.
- **Solving these problems will lead to a relatively complete and practical theory of resilient monitoring systems.**